

Low and high spin mesons from $N_f = 2$ Clover-Wilson lattices

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We present results for excited meson spectra from $N_f = 2$ clover-Wilson configurations provided by the CP-PACS Collaboration. In our study we investigate both low and high spin mesons. For spin-0 and spin-1 mesons, we are especially interested in the excited states. To access these states we construct several different interpolators from quark sources of different spatial smearings and calculate a matrix of correlators. For this matrix we then solve a generalized eigenvalue problem. For spin-2 and spin-3, we extract only the lowest lying states.

PACS numbers: 11.15.Ha

Keywords: Lattice gauge theory, hadron spectroscopy

I. INTRODUCTION

The calculation of hadron masses is one of the central subjects in lattice QCD since it gives us the opportunity to study such nonperturbative quantities from first principles. The results of such calculations (with their proper extrapolations) can then be compared directly to experiment. This allows us to clarify the internal structure of experimentally known resonances and also enables us to predict masses and properties of states which have not yet been found. Since the precise nature of many hadron resonances is unknown, lattice QCD calculations provide an indispensable contribution to their understanding.

However, this is not the only reason why hadron masses are the subject of very intensive studies in lattice QCD. A second, more technical reason is that we want to know to what extent our calculations are affected by systematic errors, which are usually connected to limited computer resources. The calculation of hadron masses gives us the possibility to study these systematics of our formulation by allowing us to compare our results directly with precise experimental measurements.

While it is well understood how to extract the mass of the ground state in a given channel, a clean extraction of the masses of excited states in a lattice QCD calculation is still a challenge. One of the main difficulties is the fact that excited states only appear as subleading exponentials in Euclidean two-point correlation functions. To extract them, a variety of approaches has been tried. They range from brute-force multiexponential fits [1] to more sophisticated techniques using Bayesian priors methods [2, 3, 4] and “NMR-inspired blackbox” methods [5]. Even evolutionary algorithms have been considered [6]. A number of these methods have been studied and compared in [7]. However, probably the most powerful technique is the variational approach [8, 9, 10, 11, 12], which is also the method we use for our studies. In this approach one studies not only a single correlator but a whole matrix of correlation functions.

To access the crucial information contained in this matrix, a rich enough basis of interpolating operators (i.e., products of creation and annihilation operators with the correct quantum numbers, which have good overlap with the hadron wave function on the lattice) has to be constructed.

For that purpose we follow a strategy which already has been very successful in quenched simulations [13, 14]: We construct quark sources of different spatial shapes and then construct a large number of interpolators from these sources. As in the quenched case, by using two different gauge covariant smearings, we can mimic radial excitations. In addition we augment our basis with “p-wave sources”, i.e., sources which should have overlap with orbital excitations.

Previous work on excited and high spin mesons in the light quark sector can be found in Refs. [15, 16, 17] for quenched calculations and in Refs. [1, 18] for the dynamical case.

Preliminary results of this investigation were presented in [19, 20]. These studies are accompanied by similar ones on quenched lattices with chirally improved fermions [21].

In addition to these excitations in the low spin sector, we perform an exploratory study of mesons with spin 2 and 3.

In the following sections, we discuss the methods which we use to create the interpolators for our simulations and for extracting the excited states. Then, after briefly describing the details of the simulations, we present our findings for the meson spectrum. We give reasons for our choices of fit ranges and the expressions we have used for the chiral extrapolations. In the end we summarize our results for these channels and compare them to experimental values. All numerical details of our results are summarized at the end of this paper.

II. THE METHOD

A. Low spin

Our calculation of the excited states of spin-0 and spin-1 mesons is based upon the variational method [8, 9]. The idea is to use several different interpolators $O_i, i = 1, \dots, N$ with the quantum numbers of the desired state and to compute all cross correlations

$$C(t)_{ij} = \langle O_i(t) \overline{O}_j(0) \rangle. \quad (1)$$

In Hilbert space these correlators have the decomposition

$$C(t)_{ij} = \sum_n \langle 0 | O_i | n \rangle \langle n | O_j^\dagger | 0 \rangle e^{-t E_n}. \quad (2)$$

Using the factorization of the amplitudes one can show [9] that the eigenvalues $\lambda^{(k)}(t)$ of the generalized eigenvalue problem

$$C(t) \vec{v}^{(k)} = \lambda^{(k)}(t) C(t_0) \vec{v}^{(k)}, \quad (3)$$

behave as

$$\lambda^{(k)}(t) \propto e^{-(t-t_0) M_k} [1 + \mathcal{O}(e^{-(t-t_0) \Delta E_k})], \quad (4)$$

where $E_k = \sqrt{\vec{p}^2 + m^2}$ is the energy of the k -th state and ΔE_k is the difference to the energy closest to E_k [22]. For a more detailed discussion of the error terms see [11, 12]. In Eq. (3) the eigenvalue problem is normalized with respect to a time slice $t_0 < t$.

In order to obtain mass spectra of states a spatial Fourier transformation is applied to the sink operator in order to project to zero momentum

$$O_i(t) = O_i(t, \vec{p}=0) = \frac{1}{V_3} \sum_{\vec{x}} O_i(t, \vec{x}) e^{i \vec{p} \cdot \vec{x}}, \quad (5)$$

with $\vec{p} = 0$.

At this point we mention two other remarkable properties of the variational method. First, it can be used to separate ghost contributions, as they appear in quenched or partially quenched calculations, from proper physical states. In the spectral decomposition (2) ghosts appear with a modified time dependence. In [10] it was shown that the ghost contribution couples to an individual eigenvalue (up to the correction term). Thus, these eigenvalues can be excluded from the analysis of the desired states. No modeling is necessary and thus no further uncertainties are introduced.

Second, the eigenvectors of the generalized eigenvalue problem (3) can be used to optimize the interval for fitting the eigenvalues. If one plots the entries of the eigenvector $\vec{v}^{(k)}$ as a function of t , one finds that they form a plateau essentially in the same interval as the effective mass. Only in the time interval where both, eigenvector components and effective mass, form a steady plateau, a fit to the eigenvalues is unambiguous. Furthermore, the eigenvectors contain information about the strength with which the different basis interpolators couple to a hadronic state. Thus, one can view them as a “fingerprint” of the corresponding state.

The variational method heavily relies on a basis of operator which has a large overlap with the states one is interested in. To construct such a basis we use several differently smeared quark sources. In a previous, quenched study [13, 14] we have optimized the smearings to match Gaussians which are ground and excited states of a spherical harmonic oscillator. However, a first study on dynamical clover-Wilson lattices has shown that it is very difficult to perform a similar matching. The reason is that with changing sea-quark mass the lattice spacing also changes which means that the smearing parameters have to be tuned for each set of configurations. In order to avoid such fine tuning procedures we simplify our construction of quark sources by using only a single Gaussian source which we generate via Jacobi smearing [23, 24]. The idea of Jacobi smearing is to create an extended source by iteratively applying the hopping part of the Wilson term (without Dirac structure) within the time slice of source and sink:

$$b^{(\alpha,a)} = S_J P^{(\alpha,a)} , \quad S_J = \sum_{n=0}^N \kappa^n H^n ,$$

$$H(\vec{x}, \vec{y}) = \sum_{i=1}^3 \left[U_i(\vec{x}) \delta_{\vec{x}+\hat{i}, \vec{y}} + U_{-i}(\vec{x}) \delta_{\vec{x}-\hat{i}, \vec{y}} \right]. \quad (6)$$

P	Point source at $x = 0$
n	Narrow source from Jacobi smearing P
L	Covariant 3D lattice Laplacian applied on n
∇_x	Covariant derivative ∇_x applied on n
∇_y	Covariant derivative ∇_y applied on n
∇_z	Covariant derivative ∇_z applied on n

TABLE I: List of the quark sources used and their specific smearing operations.

We refer to the so constructed source as *narrow source* in the following and denote it with n . In order to still allow for a radial excitation we also include a source where we apply a three-dimensional gauge covariant lattice Laplacian

$$\Delta^{(3)}(\vec{x}, \vec{y}) = \sum_{i=1}^3 \left(U_i(\vec{x}) \delta_{\vec{x}+\hat{i}, \vec{y}} + U_{-i}(\vec{x}) \delta_{\vec{x}-\hat{i}, \vec{y}} - 2\delta_{\vec{x}, \vec{y}} \right) \quad (7)$$

onto the narrow smeared sources. This one we call *Laplacian source* and denote it with L . Since both Jacobi smearing and the Laplacian are scalar operators, these do not change the quantum numbers of our generic meson interpolators.

Further operators can be added to the operator basis by also exploring the possibility of orbital excitations. To do so we include in our quark sources additional *derivative sources*. They are constructed by applying a symmetric covariant lattice derivative

$$\nabla_i(\vec{x}, \vec{y}) = \frac{1}{2} \left(U_i(\vec{x}) \delta_{\vec{x}+\hat{i}, \vec{y}} - U_{-i}(\vec{x}) \delta_{\vec{x}-\hat{i}, \vec{y}} \right) \quad (8)$$

in the appropriate direction onto the narrow smeared source. However, the resulting derivative sources, denoted by ∇_x , ∇_y , and ∇_z , have to be combined appropriately with Dirac gamma matrices, to construct meson interpolators with the desired quantum numbers. The necessary group theory for this can be found in [25] and is later also used to construct operators for high spin mesons.

Finally, we also incorporate pointlike sources, denoted by P , to our set of smearings. Although the resulting interpolators have smaller overlap with the states, these additional sources give us the opportunity to not only extract the masses of the mesons, but also to compute local matrix elements which can be related to the decay constants of the mesons.

To summarize, we use the six different quark sources listed in Table I: a point source P , a narrow smeared source n , a source L , where a covariant spatial laplacian is applied to the narrow source, and derivative sources ∇_x , ∇_y , and ∇_z . The latter ones are created by applying a covariant derivative in the corresponding spatial direction onto the narrow source. For the narrow source we use Jacobi smearing with fixed parameters ($N = 8, \kappa = 0.20$).

B. High spin

For the high spin mesons we try to extract only the ground states at the moment. Therefore, we can restrict ourselves to single correlators.

The meson interpolators we use for this purpose are taken from the paper of X. Liao and T. Manke [25] which have been already used for calculating excited charmonium states [26]. These operators contain certain combinations of Dirac γ matrices and necessarily also lattice derivative operators to be able to reach spin 2 and 3.

In discrete space-time one can only construct interpolators with definite lattice quantum numbers R^{PC} , in which R is one of the five irreducible representations of the cubic group, namely A_1, A_2, E, T_1 and T_2 . In order to determine the continuous quantum numbers J^{PC} one has to map the finite number of irreducible representations of the cubic group to the infinite number of irreducible representations of the continuous rotation group. This is complicated by the fact that this projection is not unique. The mapping from R to J for the first lowest spin states is given by

$$\begin{aligned} A_1 &\rightarrow J = 0, 4, \dots \\ A_2 &\rightarrow J = 3, \dots \\ T_1 &\rightarrow J = 1, 3, 4, \dots \\ T_2 &\rightarrow J = 2, 3, 4, \dots \\ E &\rightarrow J = 2, 4, \dots \end{aligned} \quad (9)$$

β	$L^3 \times T$	c_{SW}	$a[fm]$	$La[fm]$	κ_{sea}	#configs
1.80	$12^3 \times 24$	1.60	0.2150(22)	2.580(26)	0.1409	129
					0.1430	104
					0.1445	144
					0.1464	80
1.95	$16^3 \times 32$	1.53	0.1555(17)	2.488(27)	0.1375	113
					0.1390	137
					0.1400	137
					0.1410	99

TABLE II: Details about the CP-PACS configurations used. The values are taken from Refs. [29, 30].

Of course, for our simulations, we are especially interested in operators which transform according to T_2 and E , as well as A_2 , since the lowest continuum spins to which these couple are $J = 2$ and $J = 3$, respectively. To construct the interpolators for the high spin states we again combine one or two gauge covariant lattice derivatives [see Eq. 8] with appropriate Dirac gamma matrices according to Ref. [26]. Also, here we want to improve the overlap with the physical states. Therefore, we apply to each quark source a gauge invariant Gaussian smearing using a spatial width of $2.4a$ and 16 iterations. However, the correlators in these channels turn out to be particularly noisy. To further improve our results we use APE smeared links ($\alpha = 2.5$ and $N = 15$), but only to create the source and the sink meson interpolators.

III. SIMULATION DETAILS

We calculate our meson correlators on configurations with two flavors of dynamical quarks. These configurations have been generated by the CP-PACS Collaboration using clover-Wilson fermions [27] with a mean field improved clover coefficient and an RG improved gauge action [28].

In Table II, we summarize details of the configurations used in our simulations. For each lattice size, there exist four ensembles with different sea-quark mass. The values κ_{sea} have been chosen in such a way that the ratio m_{PS}/m_V is approximately the same for the different lattice sizes. For our simulations we use only every fifth available configuration in each ensemble in order to reduce effects coming from finite autocorrelation times and at the same time save computational resources. Thus, we analyze between 80 and 150 configurations per ensemble for our calculations (see Table II). There exists also an even finer lattice with $24^3 \times 48$ sites, but we lacked the computer time to make use of it. More information about these configurations, especially on how they have been generated can be found in Refs. [29, 30].

For our simulations we used Chroma [31]. At the time this project started only version 2.2.1 of this lattice QCD library was available, which did not contain a proper implementation of clover-Wilson fermions, Jacobi smearing, and the calculation of cross-correlation matrices. Therefore, we developed our own routines for these tasks. Starting with version 3 of Chroma, the above-mentioned routines were also implemented. For consistency, we stick to our own routines for calculating excited states in the low spin sector, while using native Chroma with appropriate XML input files for the high spin mesons. Thus, for the high spin sector, we can take advantage of different optimizations, like the SSE optimized Wilson Dslash [32] and Peter Boyle's BAGEL [33] for running on QCDOC [34].

IV. RESULTS

A. Low spin

1. Effective masses

In the following we present the results of our calculations. For our analysis we take advantage of several symmetries of the cross-correlation matrix. We find that the matrices $C(t)$ are real and symmetric within error bars. Therefore, we symmetrize them by replacing $C_{ij}(t)$ with $(C_{ij}(t) + C_{ji}(t))/2$. We can increase our statistics even further by taking into account the contributions which are proportional to $\exp[-(T - t)M_n]$. We symmetrize our correlators by replacing $C(t)$ with $(|C(t)| + |C(T - t)|)/2$ and use the resulting matrix in the variational method.

The eigenvalues, which we obtain from the generalized eigenvalue problem (3), can then be fitted to the function

$$\lambda^{(k)}(t, t_0) = Ae^{-M_k(t-t_0)}, \quad (10)$$

where we use $t_0 = 1$ in all cases. Ideally one should use the largest possible value of t_0 ; however, due to the limited signal for the excited states, we are not able to go to larger values. To determine fit ranges, we define the effective mass

$$m_{eff}(t + \frac{1}{2}) = \ln \left(\frac{\lambda^{(k)}(t, t_0)}{\lambda^{(k)}(t + 1, t_0)} \right). \quad (11)$$

This quantity should form a plateau as a function of t once the contributions of the higher excited states are strongly suppressed. Additional information is provided by the eigenvectors $\vec{v}^{(k)}$. Their components should also show a plateau when only a single state contributes.

Another important feature of our analysis is that we use only a submatrix of the correlator matrix. We refer to this procedure as *pruning of the operator basis*. The reason is that many of the interpolators have only small overlap with the physical state or, given the limited number of gauge configurations, they convey no new information. Their inclusion contributes mainly noise to the correlator. Also, one can show [35] that choosing certain interpolator combinations helps in suppressing contributions of higher order corrections in the different eigenvalues. In this way, one can improve the effective mass plateaus to a certain extent by choosing an optimal, and often smaller, interpolator basis. However, to find such a combination is rather difficult, since the number of possibilities to choose a certain interpolator combination is extremely high. Of course we cannot check all possible combinations; however, one quickly learns which operators are worth leaving out. This might also make pruning very subjective and thus can lead to ambiguous results if several combinations of operators seem to be equally good but give slightly different effective mass plateaus. As long as these deviations are well within errors we should be allowed to choose anyone of these combinations. We choose those that appear to give the best effective mass plateaus from the largest possible number of eigenvalues (here, 2 for the pseudoscalars and vectors and 1 for the rest).

In Fig. 1, we show the effective masses for pseudoscalar (PS), scalar (SC), vector (V), and axialvector (AV) mesons obtained on the $12^3 \times 24$ lattice for the four quark masses we have used in our calculations. The horizontal lines denote the time intervals where we have performed correlated fits to the eigenvalues and represent the resulting masses and their statistical errors.

We obtain excellent plateaus for the pseudoscalar and vector ground states. For these channels we are also able to extract first excited states. There, however, the results are not that good: The plateaus consist of only two or three effective mass points and are very noisy. We find that the ground states for both meson channels are practically unaffected by the choice of operators. For the excited pseudoscalar meson we are able to use the same optimal interpolator combination for all quark masses. However, to obtain results for the excited vector meson state we have to alter the optimal operator combination for each sea-quark mass (see Table III).

The results for scalar and axialvector are also very good, however, slightly noisier than those of pseudoscalar and vector ground states. The fact that the pseudoscalar and vector channels yield better results than the other mesons is usually observed in lattice QCD. This is not unexpected since these states are much lighter than all the others and thus yield a better signal for a larger number of time slices.

In Fig. 2, we present the effective masses from the finer lattice.

Again we obtain excellent results for pseudoscalar and vector ground states with long clear plateaus. However, the situation for the excited pseudoscalar and vector states improved only marginally. The plateaus are noisy and rather short; often we can include only three or four time slices in our fits. Certainly an improvement is given by the fact that for the finer lattice we can choose the same optimal combination for all sea-quark masses, except for the smallest quark mass. There we altered the optimal interpolator combination for the pseudoscalar meson slightly (see Table IV).

In the scalar and axialvector channel we find only a slight improvement when going to the finer lattice. For the scalar meson it is necessary to choose a different operator for $\kappa = 0.1400$ than for the other masses. Since the combination $L\bar{l}n$ is very similar to $\nabla_i \bar{l} \nabla_i$ (both of them represent a three-dimensional lattice Laplacian but with different displacement), we do not regard this as a problem.

Fortunately, in our previous quenched studies [13, 14], we were able to use for each valence quark mass the same time slice as the starting point of the fit intervals. In this study, however, we sometimes need to change this time slice as we move from one quark mass to next one. The reason is that the ensembles for different sea-quark masses are generated independently. Thus, they should be completely uncorrelated, in contrast to the quenched case, where we changed only valence quark mass but always used the same set of configurations. Additionally, the effective lattice spacing depends on the sea-quark mass. Nevertheless, we still require that both the effective mass and components of the corresponding eigenvector show plateaus in the fit interval.

The numerical results of our correlated fits together with the optimal operators for the meson states can be found in Tables III and IV.

2. Pseudoscalar meson ground state

For the pion ground state the results of our fits are presented in Fig. 3, where we plot the pion mass squared as a function of κ^{-1} . To be able to extrapolate our other results to the chiral limit, we have to determine the critical quark mass. It is defined as the value κ_c^{-1} where the mass of the pseudoscalar meson vanishes.

For the pseudoscalar meson the appropriate chiral extrapolation formula is given by the resummed Wilson chiral perturbation theory (RW χ PT) [29]. It reads

$$m_{PS}^2 = Am \left[-\log \left(\frac{Am}{\Lambda_0^2} \right) \right]^{\omega_0} \left[1 + \omega_1 m \log \left(\frac{Am}{\Lambda_3^2} \right) \right], \quad (12)$$

where $m = \frac{1}{2}(\kappa^{-1} - \kappa_c^{-1})$ is the quark mass and A , Λ_0 , Λ_3 , ω_0 , and ω_1 are parameters in the theory. Since we have only four data points for each lattice it is not possible to use this expression as a fit function. Therefore, we restrict ourselves to a much simpler function given by

$$(am_{PS})^2 = B_{PS} m + C_{PS} m^2, \quad (13)$$

and we take κ_c^{-1} as an additional fit parameter. The linear term is motivated by Wilson χ PT without resummation

$$m_{PS}^2 = Am \left[1 + \omega_1 m \log \left(\frac{Am}{\Lambda_3^2} \right) + \omega_0 \log \left(\frac{Am}{\Lambda_0^2} \right) \right], \quad (14)$$

while we include the quadratic term in order to account for the slight curvature of our results. Since we are working at pion masses from approximately 500 MeV to 1 GeV, it is highly questionable to what extent χ PT is applicable.

3. Vector meson ground state

In the upper two plots of Fig. 4, we present our results for the vector meson ground state as a function of the mass of the pseudoscalar ground state.

For the chiral extrapolations we use

$$am_V = A_V + B_V(am_{PS})^2 + C_V(am_{PS})^4 \quad (15)$$

as the fit function.

Our results for the pion and rho ground states are slightly different from the ones obtained by the CP-PACS Collaboration. For consistency, we thus redetermine the physical point and the lattice spacing by following the procedure described in [30]. For the physical point, we consider the ratio

$$\frac{am_\pi}{A_V + B_V(am_\pi)^2 + C_V(am_\pi)^4} = \frac{M_\pi}{M_\rho}, \quad (16)$$

where $M_\pi = 0.1396$ GeV and $M_\rho = 0.7755$ GeV are fixed to the experimental values. The lattice spacing is then given by

$$a = \frac{am_\rho}{M_\rho}, \quad (17)$$

with $am_\rho = am_V(am_\pi)$ being the mass of the rho meson in lattice units determined at the physical point for am_π , determined via Eq. (16). In addition, we can also compute κ_{ud}^{-1} which corresponds to up/down quark mass on the lattices by solving

$$(am_{PS})^2(\kappa_{ud}) = (am_\pi)^2. \quad (18)$$

The resulting values for the physical point am_π , the lattice spacing a , and the parameters am_π and am_ρ are summarized in Table VI.

4. Scalar and axialvector meson ground state

After determining the physical point and the lattice spacing, we can discuss the results for the other meson channels.

We start with the scalar ground state which is shown in the second row of plots in Fig. 4. For the $12^3 \times 24$ lattice, we find that the scalar mass depends linearly on the squared pion mass. Therefore, we perform linear fits in $(am_{PS})^2$ for the chiral extrapolation. This means that we fit our results to

$$am_{SC} = A_{SC} + B_{SC}(am_{PS})^2. \quad (19)$$

However, for the finer lattice the scalar meson mass for the smallest quark mass shows some deviation from the linear behavior of the other points. Therefore, we extend the expression in Eq. (19) by an additional term $C_{SC}(am_{PS})^4$. We also try to add such an additional term to the fit functions of the other meson states. However, in all these cases the fit results for the corresponding parameter C is consistent with zero.

For the axialvector meson ground state (see Fig. 4 lower plots) we find that the results on both lattices depend linearly on $(am_{PS})^2$. Thus, we use

$$am_{AV} = A_{AV} + B_{AV}(am_{PS})^2 \quad (20)$$

as the fit function for our chiral extrapolations. The only point which shows a slight deviation from a linear behavior is the point at $(am_{PS})^2 \approx 0.53$. Nevertheless, we have decided to include this point in our fit, since leaving it out changes our results negligibly.

5. Pseudoscalar and vector meson excited state

We start our discussion of the excited states with the excited pseudoscalar meson. In the upper plots in Fig. 5 we plot the results of our fits to the eigenvalues as a function of $(am_{PS})^2$. On both lattices we find a linear behavior except for the smallest quark mass on the finer lattice where the computed mass lies exceptionally high. We therefore exclude this point in our chiral extrapolation [including the point changes the fit results to $A = 1.47(8)$, $B = 0.44(12)$ with $\chi^2/d.o.f. = 3.42$].

Next, we discuss the results for the excited vector meson channel which are shown in the lower plots of Fig. 5. We find that our results on the coarse lattice are somewhat problematic. We observe a very jumpy behavior of the meson masses as a function of $(am_{PS})^2$. A reason for this might be that we had to choose different operator combinations for the different sea-quark masses. This also makes the chiral extrapolation very difficult. We try a linear fit as the simplest choice. This leads to a value of $\chi^2/d.o.f. \approx 4$ which shows that the fit is not reliable. Thus, the result should not be taken too seriously. On the finer lattice, we again find that the result for the smallest quark mass lies exceptionally high. Thus, we exclude also this point in our chiral extrapolation [including the point in our fit changes the results to $A = 1.58(7)$, $B = 0.59(11)$ with $\chi^2/d.o.f. = 1.87$]. Since the operators we use for the vectors couple also to a state which has $J = 3$ in the continuum, we cannot exclude the possibility that we are seeing a ρ_3 meson here. However, our results for operators with a *minimum* J of 3 give an even higher mass (see below).

B. High spin

1. Effective masses

Next we present our results for high spin and exotic mesons, where we used single correlation functions to extract ground state masses. To select appropriate ranges where we can fit a single exponential function of the form $A_0 e^{-m_0 t}$ we use effective mass plots from the folded correlators.

Figure 6 shows some selected plots from our coarse lattice, where we obtain signals for most of our operators coupling to spin $J = 2$.

For the a_2 meson which has quantum numbers $J^{PC} = 2^{++}$ there are three operators available (see Ref. [25]). Masses and fitting ranges for the interpolators $\rho \times \nabla_- T_2$ and $a_1 \times D_- E$ are shown in Fig. 6. One can observe some short plateaus in time ranges $t = 1 - 5$ and $2 - 5$ which are of a different quality for the various operators. Also the effective masses are not always consistent within the errors for the different operators, even though they should couple to the same state. Therefore, we fitted the two lowest lying plateaus belonging to the interpolators $\rho \times \nabla_- T_2$ and $a_1 \times D_- E$.

Liao and Manke [25] also provide three operators that couple to $J^{PC} = 2^{--}$. Their signal is weaker, the errors are somewhat bigger and it is often tricky to find appropriate plateaus. One may, for example, look at the plots for the interpolator $\rho \times D \cdot T_2$ and $a_1 \times \nabla \cdot T_2$ in Fig. 6. However, we tried to fit the $\rho \times D \cdot T_2$ in a range $t = 2 - 4$.

Our results for the π_2 meson which has quantum numbers $J^{PC} = 2^{-+}$ are quite poor and noisy on the coarse lattice, so that we could not detect considerable plateaus.

Our correlated fit results for the interpolators which provide feasible signals on this coarse lattice are listed in Table VII.

In Fig. 7, one can see example plots for interpolators with quantum numbers $J^{PC} = 2^{++}$ and 2^{--} from our $16^3 \times 32$ lattice.

Let us now have a closer look at these mesons on the finer lattice. For the a_2 meson we again plot the effective masses for the operators $\rho \times \nabla \cdot T_2$ and $a_1 \times D \cdot E$. The plateaus have become longer and clearer but the discrepancies between the masses of the different interpolators have increased. For short distances $t < 4$ the plateaus for the $a_1 \times D \cdot E$ and the $a_1 \times D \cdot T_2$ are lying much higher than for the $\rho \times \nabla \cdot T_2$. Only for times $t > 5$ the masses become lower and agree within the errors, but simultaneously, the signal becomes very noisy. Thus, we think that the $\rho \times \nabla \cdot T_2$ has more overlap with the ground state than the other two operators, which might be contaminated strongly by excited states which then become suppressed only for large times. Nevertheless, even for the $\rho \times \nabla \cdot T_2$ it is not unambiguous where to start fitting. This is why we present results from different fit ranges for this interpolator.

The situation for the ρ_2 state is similar to that of the a_2 . In comparison to our coarse lattice one can observe reduced error bars, along with longer and clearer plateaus which are mostly consistent within the errors. Only the mass of the $\rho \times D \cdot T_2$ is somewhat higher. Moreover, there seems to be a step in the effective mass from the time slice 4 to 5 for this interpolator. For this reason we fitted only the interpolators $a_1 \times \nabla \cdot T_2$ and $a_1 \times \nabla \cdot E$ for times $t > 2$.

We also observe very weak signals and huge errors for the π_2 meson on the $16^3 \times 32$ lattice. However, we tried to fit this state in time ranges $t = 2 - 4$.

Even for some high spin mesons with spin quantum number $J = 3$ and for the exotic π_1 state appropriate plateaus have been detected. Although their masses are quite high and the plateaus are really short, we present results for the interpolators $a_1 \times D \cdot A_2$, $\rho \times D \cdot A_2$ and $b_1 \times \nabla \cdot T_1$ in time ranges $t = 1 - 4$ and $2 - 4$.

All our correlated fit results for the interpolators which provide sufficiently stable signals on this finer lattice are collected in Table VIII.

2. Chiral extrapolation

For high spin and exotic meson states there are no results from chiral perturbation theory. However, we find that our masses depend almost linearly on $(am_\pi)^2$. Therefore, we perform fits of the form

$$am_{HS} = A_{HS} + B_{HS}(am_{PS})^2 \quad (21)$$

to the physical point. The values for the physical point am_π , and the lattice spacing a are listed in Table VI. Our extrapolation results are then summarized in Table IX.

We first discuss the results for the a_2 meson channel which are shown in Fig. 8. Here we notice a nice linear behavior of the operator $\rho \times \nabla \cdot T_2$ with small errors on the coarse lattice. On the fine lattice however, we have two data sets belonging to two different ranges in the effective mass. We fit these sets separately and average in the end, which introduces a possibly large systematic error. For the $a_1 \times D \cdot E$ we only have three data points on the coarse lattice available. Nevertheless, we try a linear fit. This however, leads to higher errors and a small value of $\chi^2/d.o.f. = 0.12$.

In Fig. 9 we present our extrapolation results for the ρ_2 meson. For the $\rho \times D \cdot T_2$ we also only have three data points on the coarse lattice. Additionally they show a quite jumpy behavior of the meson mass as a function of $(am_{PS})^2$. This might be caused by the noisy signal on that lattice and the brevity of the plateaus. Therefore, one should not trust this result too much. Here the situation becomes better on the fine lattice where the interpolator $a_1 \times \nabla \cdot T_2$ shows a good linear behavior which leads to a reliable fit with small errors. For the $a_1 \times \nabla \cdot E$ the masses are more jumpys and therefore, we obtain a quite high value of $\chi^2/d.o.f. \approx 6$, although it agrees with the result of the interpolator $a_1 \times \nabla \cdot T_2$.

Finally we discuss the results for the π_2, a_3, ρ_3 and the exotic π_1 meson state which are shown in Fig. 10 from top left to down right. For the operators $\pi \times D \cdot T_2$ and $a_1 \times D \cdot A_2$ we again have only three data points available. Therefore, the value $\chi^2/d.o.f. < 0.5$. For the operator $\rho \times D \cdot A_2$ we find a good linear behavior and obtain small errors. The last plot shows the extrapolation of the interpolator $b_1 \times \nabla \cdot T_1$ where we notice a small outlier of the linear behavior at $(am_{PS})^2 \approx 0.53$. However, we also include this point into our fit because the masses are afflicted with large errors.

V. DISCUSSION

A. Meson masses

We compute the meson spectrum by evaluating the results of the chiral extrapolations at the physical point am_π and then converting them into physical units by using our results for the lattice spacing a (see Table VI). This means that for each meson channel we calculate

$$M_{\text{meson}} = \frac{[am_{\text{meson}}(am_\pi; A_{\text{meson}}, B_{\text{meson}}, C_{\text{meson}})]}{a}, \quad (22)$$

where A_{meson} , B_{meson} , and C_{meson} are the parameters that we have obtained from our chiral extrapolations and the a in the denominator stands for the lattice spacing, which we have determined with the rho meson.

Our final results for the low spin meson spectrum are summarized in Fig. 11, where we plot our results for both lattices in comparison with the experimental values from [36]. We do not show the vector meson ground state results since they have been used to determine the lattice spacing.

For the excited pseudoscalar meson our findings are in good agreement with the $\pi(1300)$ although the error for the finer lattice is quite large, thus making it also compatible with the $\pi(1800)$.

The results for the excited vector meson lie much too high. A reason for this might be the following: Our correlators are rather noisy, i.e., our effective mass plateaus are short, thus it might be that we start our fits too early. Another explanation is that our quark masses are too large and a more sophisticated extrapolation is needed. Unfortunately this is not possible since we have too few data points. We also want to mention here that we have found something similar in our previous quenched studies [13] on a coarse lattice. There, a finer lattice was needed to obtain better results.

For the scalar meson our results on the coarse lattice are compatible with the $a_0(1450)$. However, on the finer lattice we find smaller values. The linear extrapolated results lie between the $a_0(980)$ and the $a_0(1450)$. When a quadratic fit is used, the average value for the mass becomes smaller but the error is much larger. The first finding is similar to what we already have observed in previous quenched studies with approximate chiral fermions. First studies with dynamical CI-fermions [37] however obtain a value which is consistent with the $a_0(980)$. This suggests that chiral sea quarks play a crucial role for scalar mesons.

For the axialvector meson our results are also higher than expected. They lie right between the $a_1(1260)$ and the $a_1(1640)$. This is similar to what we have seen in our previous quenched studies. Probably, here chiral sea quarks are needed to improve the situation, too.

Our final results for the meson spectrum of high spin and exotic states are summarized in Fig. 12, where we again plot our results for both lattices in comparison to the experimental values from [36].

Our results for the a_2 meson lie between the $a_2(1320)$ and the $a_2(1700)$ which is higher than expected. The effective masses for the discussed interpolators are quite short on the coarse lattice and also do not agree within the errors for the various operators on both lattices. Therefore, it might be that the operators we used have only poor overlap with the physical ground state and we start our fits too early. For this reason finer lattices and a more advanced analysis, as it was done for the low spin mesons, would be needed to improve our results. But also our usage of quite large quark masses, as mentioned above, might affect this shift.

For the ρ_2 meson we observe only weak signals and very short plateaus on the coarse lattice for one of our operators coupling to that state. Thus we obtain quite large errors for our result on that lattice. However, on the fine lattice we find very clear signals for that state and our results agree within the errors and the physical ground state $\rho_2(1940)$.

For the π_2 meson we obtain only weak and noisy signals on the fine lattice. Hence, our result is afflicted with huge errors and lies too high.

We also found short effective mass plateaus for the high spin states a_3 , ρ_3 and the exotic meson π_1 . The extrapolation, however, leads to masses much larger than those found for the experimental ground states. One possible explanation for these findings might be finite volume effects, since these states should have more extended wave functions. In this case, larger and finer lattices would be needed to obtain longer and clearer plateaus and to reduce discretization and finite volume effects.

B. Possible systematics

Since we work at pion masses above 500 MeV a number of hadronic decay channels which would normally be open are suppressed. This introduces systematic shifts to our observed meson spectrum. Only by going to much smaller quark masses and taking into account explicit mixing with multiparticle states can we resolve these issues (see Refs. [38, 39] for hybrid meson decays and Refs. [40, 41, 42] for rho meson decay).

$\kappa = \kappa_{sea} = \kappa_{val}$	am	$[t_{min}, t_{max}]$	$\chi^2/d.o.f.$	Optimal operators
Pseudoscalar ground state				
0.1409	1.1520(23)	[5,10]	0.16	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1430	0.9774(28)	[2,10]	1.50	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1445	0.8201(29)	[2,9]	1.10	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1464	0.5363(60)	[2,9]	0.99	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
Vector ground state				
0.1409	1.4469(55)	[4,10]	0.88	$P\gamma_i P, L\gamma_i L$
0.1430	1.3070(62)	[3,10]	0.37	$n\gamma_i n, \nabla_i n$
0.1445	1.1870(57)	[2,9]	0.35	$P\gamma_i P, P\gamma_i \gamma_4 P, n\gamma_i n, L\gamma_i L$
0.1464	0.973(15)	[3,9]	0.41	$P\gamma_i P, n\gamma_i n$
Scalar ground state				
0.1409	2.188(28)	[2,7]	0.17	$\nabla_i \mathbb{1} \nabla_i$
0.1430	1.964(30)	[2,5]	0.44	$\nabla_i \mathbb{1} \nabla_i$
0.1445	1.824(29)	[2,6]	0.84	$\nabla_i \mathbb{1} \nabla_i$
0.1464	1.620(55)	[2,5]	0.24	$\nabla_i \mathbb{1} \nabla_i$
Axialvector ground state				
0.1409	2.291(51)	[3,6]	0.12	$\nabla_i \gamma_k \gamma_5 \nabla_i$
0.1430	2.022(23)	[2,6]	0.14	$\nabla_i \gamma_k \gamma_5 \nabla_i$
0.1445	1.922(21)	[2,6]	0.94	$\nabla_i \gamma_k \gamma_5 \nabla_i$
0.1464	1.651(66)	[3,6]	0.39	$\nabla_i \gamma_k \gamma_5 \nabla_i$
Pseudoscalar 1st excited state				
0.1409	2.276(40)	[2,5]	0.24	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1430	2.003(90)	[2,5]	0.16	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1445	1.868(62)	[2,4]	0.01	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1464	1.56(14)	[2,4]	0.71	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
Vector 1st excited state				
0.1409	2.436(50)	[3,5]	0.01	$P\gamma_i P, L\gamma_i L$
0.1430	2.35(13)	[2,4]	0.20	$n\gamma_i n, \nabla_i n$
0.1445	2.082(48)	[2,5]	0.53	$P\gamma_i P, P\gamma_i \gamma_4 P, n\gamma_i n, L\gamma_i L$
0.1464	2.128(42)	[2,4]	0.10	$P\gamma_i P, n\gamma_i n$

TABLE III: Results of the meson masses from the $12^3 \times 24$ lattice. The interval $[t_{min}, t_{max}]$ denotes the time range where we have fitted the eigenvalues. $\chi^2/d.o.f.$ represents the quality of our correlated fits. In the last column we show our final choice for the optimal operator combination for each meson channel.

The lattices we use are about 2.5fm in spatial extent. This may be exceptionally small for most of the excited mesons we study and may explain why many of our results come out too high.

We also use rather coarse lattices of $a = 0.2$ fm and 0.15 fm, making it difficult to unambiguously resolve the high masses of the excited states.

It has been argued recently that there is a restoration of chiral symmetry in highly excited hadrons [43]. Such considerations suggest that the use of (at least approximately) chiral fermions is important for lattice studies of excited states. Recent efforts with dynamical chirally improved fermions [44] have appeared and work for the excited meson spectrum is in progress.

Acknowledgments

We thank Christof Gattringer and Christian B. Lang for interesting discussions. The calculations were performed on our local compute cluster and the QCDOC in Regensburg. We thank Tilo Wettig for supplying the machine and Stefan Solbrig for his help and technical support. We are also indebted to the CP-PACS Collaboration for sharing their configurations with us. This work is supported in part by DFG (SFB-TR55) and GSI (RSCHAE). T.B. acknowledges previous support by the DOE(DE-FC02-01ER41183) and NSF(NSF-PHY-0555243).

$\kappa = \kappa_{sea} = \kappa_{val}$	am	$[t_{min}, t_{max}]$	$\chi^2/d.o.f.$	Optimal operators
Pseudoscalar ground state				
0.1375	0.8917(24)	[4,13]	0.58	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1390	0.7252(23)	[3,13]	0.99	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1400	0.5958(22)	[5,11]	0.88	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1410	0.4290(29)	[5,9]	0.81	$P\gamma_5 P, n\gamma_5 n$
Vector ground state				
0.1375	1.1066(35)	[4,13]	1.02	$P\gamma_i P, n\gamma_i n$
0.1390	0.9648(48)	[4,12]	0.80	$P\gamma_i P, n\gamma_i n$
0.1400	0.8611(64)	[5,12]	0.84	$P\gamma_i P, n\gamma_i n$
0.1410	0.7332(82)	[5,13]	0.40	$P\gamma_i P, n\gamma_i n$
Scalar ground state				
0.1375	1.583(41)	[3,7]	0.43	$L\mathbb{1}n$
0.1390	1.379(24)	[2,8]	0.36	$L\mathbb{1}n$
0.1400	1.263(34)	[4,8]	0.17	$\nabla_i \mathbb{1} \nabla_i$
0.1410	0.948(75)	[4,7]	1.12	$L\mathbb{1}n$
Axialvector ground state				
0.1375	1.621(19)	[2,7]	0.99	$P\gamma_i \gamma_5 P, n\gamma_i \gamma_5 L$
0.1390	1.334(74)	[5,8]	0.12	$P\gamma_i \gamma_5 P, n\gamma_i \gamma_5 L$
0.1400	1.307(48)	[4,8]	0.24	$P\gamma_i \gamma_5 P, n\gamma_i \gamma_5 L$
0.1410	1.199(28)	[3,7]	0.39	$P\gamma_i \gamma_5 P, n\gamma_i \gamma_5 L$
Pseudoscalar 1st excited state				
0.1375	1.838(30)	[2,6]	0.34	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1390	1.605(62)	[3,6]	0.37	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1400	1.46(12)	[4,6]	0.01	$P\gamma_5 P, n\gamma_5 n, \nabla_i \gamma_5 \nabla_i$
0.1410	1.660(74)	[3,6]	0.03	$P\gamma_5 P, n\gamma_5 n$
Vector 1st excited state				
0.1375	2.060(26)	[3,5]	0.08	$P\gamma_i P, n\gamma_i n$
0.1390	1.879(55)	[4,7]	0.13	$P\gamma_i P, n\gamma_i n$
0.1400	1.724(59)	[4,7]	0.23	$P\gamma_i P, n\gamma_i n$
0.1410	1.827(90)	[4,6]	0.47	$P\gamma_i P, n\gamma_i n$

TABLE IV: The same as in Table III but for the $16^3 \times 32$ lattice.

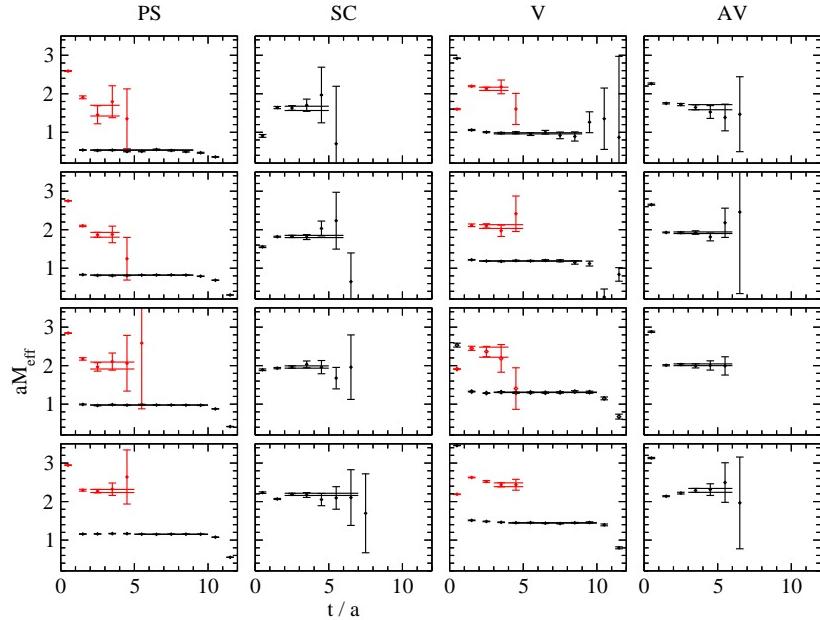
Pseudoscalar ground state				
$L^3 \times T$	κ_c^{-1}	B	C	$\chi^2/d.o.f.$
$12^3 \times 24$	6.7678(28)	9.44(25)	-8.4(1.4)	0.10
$16^3 \times 32$	7.0366(22)	6.61(24)	1.0(1.6)	0.36
$L^3 \times T$				
A	B	C	$\chi^2/d.o.f.$	
Vector ground state				
$12^3 \times 24$	0.801(29)	0.656(66)	-0.128(35)	0.78
$16^3 \times 32$	0.586(18)	0.857(74)	-0.255(67)	0.37
Scalar ground state				
$12^3 \times 24$	1.452(48)	0.549(49)	—	0.14
$16^3 \times 32$	0.927(57)	0.85(11)	—	2.28
$16^3 \times 32$	0.73(14)	1.65(55)	-0.75(51)	2.41
Axialvector ground state				
$12^3 \times 24$	1.546(54)	0.528(63)	—	1.91
$16^3 \times 32$	1.064(24)	0.696(53)	—	0.88
Pseudoscalar 1st excited state				
$12^3 \times 24$	1.41(10)	0.652(32)	—	0.14
$16^3 \times 32$	1.15(15)	0.86(20)	—	0.001
Vector 1st excited state				
$12^3 \times 24$	1.988(52)	0.309(63)	—	3.94
$16^3 \times 32$	1.473(94)	0.74(13)	—	0.13

TABLE V: Numerical results of the chiral extrapolations of the different meson channels in Sec. IV A.

$L^3 \times T$	κ_{ud}^{-1}	am_π	am_ρ	$a[\text{fm}]$
$12^3 \times 24$	6.7722(27)	0.1438(28)	0.814(52)	0.2071(132)
$16^3 \times 32$	7.0400(21)	0.1055(18)	0.595(32)	0.1515(82)

TABLE VI: Results for the redetermination of the physical point and the lattice spacing. In addition, the bare quark mass parameter κ_{ud}^{-1} which corresponds to the mass of the up/down quark has been computed.

$\kappa = \kappa_{sea} = \kappa_{val}$	am	$[t_{min}, t_{max}]$	$\chi^2/d.o.f.$	ops
$J^{PC} = 2^{++}$				
0.1409	2.148(15)	[1, 5]	3.79	$\rho \times \nabla_- T_2$
0.1430	1.963(19)	[1, 6]	0.22	$\rho \times \nabla_- T_2$
0.1445	1.844(16)	[1, 4]	1.36	$\rho \times \nabla_- T_2$
0.1464	1.689(22)	[1, 4]	1.42	$\rho \times \nabla_- T_2$
$J^{PC} = 2^{++}$				
0.1409	2.228(41)	[2, 4]	0.96	$a_1 \times D_- E$
0.1430	2.070(51)	[2, 4]	0.08	$a_1 \times D_- E$
0.1445	1.912(42)	[2, 5]	0.08	$a_1 \times D_- E$
0.1464	—	—	—	$a_1 \times D_- E$
$J^{PC} = 2^{--}$				
0.1409	—	—	—	$a_1 \times \nabla_- T_2$
0.1430	2.487(121)	[2, 4]	0.10	$a_1 \times \nabla_- T_2$
0.1445	2.360(111)	[2, 4]	0.002	$a_1 \times \nabla_- T_2$
0.1464	—	—	—	$a_1 \times \nabla_- T_2$
$J^{PC} = 2^{--}$				
0.1409	—	—	—	$a_1 \times \nabla_- E$
0.1430	—	—	—	$a_1 \times \nabla_- E$
0.1445	2.138(32)	[1, 4]	0.21	$a_1 \times \nabla_- E$
0.1464	1.922(45)	[1, 4]	0.46	$a_1 \times \nabla_- E$
$J^{PC} = 2^{--}$				
0.1409	—	—	—	$\rho \times D_- T_2$
0.1430	2.677(173)	[2, 4]	0.03	$\rho \times D_- T_2$
0.1445	2.193(107)	[2, 4]	0.64	$\rho \times D_- T_2$
0.1464	2.139(192)	[2, 4]	0.20	$\rho \times D_- T_2$

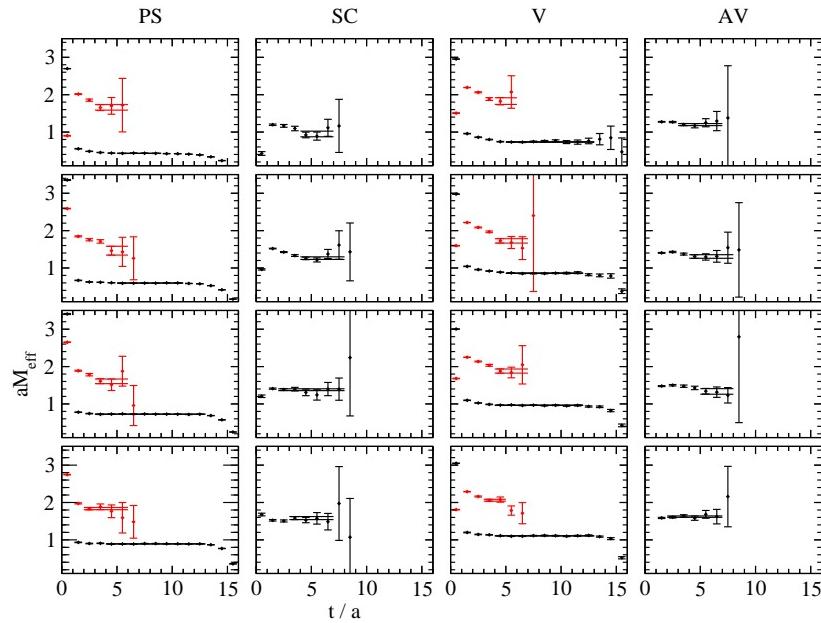
TABLE VII: Meson masses from the $12^3 \times 24$ lattice.FIG. 1: Effective mass plots for mesons from our coarse lattice (with $\kappa = 0.1464, 0.1445, 0.1430, 0.1409$ from top to bottom). Both ground and excited states are shown, along with the $M \pm \sigma_M$ results (horizontal lines) from correlated fits to the corresponding time intervals. For the PS channel, we show results for both operator combinations.

$\kappa = \kappa_{sea} = \kappa_{val}$	am	$[t_{min}, t_{max}]$	$\chi^2/d.o.f.$	ops
$J^{PC} = 2^{++}$				
0.1375	1.648(14)	[2, 8]	1.64	$\rho \times \nabla_- T_2$
0.1390	1.554(14)	[2, 7]	0.93	$\rho \times \nabla_- T_2$
0.1400	1.481(14)	[2, 4]	0.93	$\rho \times \nabla_- T_2$
0.1410	1.384(16)	[2, 7]	0.54	$\rho \times \nabla_- T_2$
0.1375	1.565(35)	[4, 8]	0.59	$\rho \times \nabla_- T_2$
0.1390	1.489(37)	[4, 7]	0.07	$\rho \times \nabla_- T_2$
0.1400	1.389(35)	[4, 7]	2.43	$\rho \times \nabla_- T_2$
0.1410	1.317(61)	[4, 7]	0.22	$\rho \times \nabla_- T_2$
$J^{PC} = 2^{--}$				
0.1375	1.952(25)	[2, 7]	0.24	$a_1 \times \nabla_- T_2$
0.1390	1.845(22)	[2, 6]	0.25	$a_1 \times \nabla_- T_2$
0.1400	1.726(21)	[2, 6]	0.69	$a_1 \times \nabla_- T_2$
0.1410	1.600(20)	[2, 4]	0.92	$a_1 \times \nabla_- T_2$
$J^{PC} = 2^{-+}$				
0.1375	1.971(27)	[2, 5]	0.48	$a_1 \times \nabla_- E$
0.1390	1.866(23)	[2, 6]	0.23	$a_1 \times \nabla_- E$
0.1400	1.757(25)	[2, 5]	0.88	$a_1 \times \nabla_- E$
0.1410	1.551(24)	[2, 5]	0.07	$a_1 \times \nabla_- E$
$J^{PC} = 3^{++}$				
0.1375	2.039(99)	[2, 4]	0.06	$\pi \times D_- T_2$
0.1390	1.964(122)	[2, 4]	0.25	$\pi \times D_- T_2$
0.1400	1.802(144)	[2, 4]	0.26	$\pi \times D_- T_2$
0.1410	—	—	—	$\pi \times D_- T_2$
$J^{PC} = 3^{--}$				
0.1375	2.319(84)	[2, 4]	1.11	$a_1 \times D_- A_2$
0.1390	2.107(75)	[2, 4]	0.04	$a_1 \times D_- A_2$
0.1400	—	—	—	$a_1 \times D_- A_2$
0.1410	1.975(72)	[2, 4]	0.02	$a_1 \times D_- A_2$
$J^{PC} = 1^{-+}$				
0.1375	2.127(71)	[2, 4]	0.0008	$b_1 \times \nabla_- T_1$
0.1390	2.196(71)	[2, 4]	0.25	$b_1 \times \nabla_- T_1$
0.1400	1.952(61)	[2, 4]	0.80	$b_1 \times \nabla_- T_1$
0.1410	1.908(79)	[2, 4]	0.85	$b_1 \times \nabla_- T_1$

TABLE VIII: Meson masses from the $16^3 \times 32$ lattice.

$L^3 \times T$	A	B	$\chi^2/d.o.f.$
$2^{++} = \rho \times \nabla_- T_2$			
$12^3 \times 24$	1.549(23)	0.446(24)	0.49
$16^3 \times 32$	1.322(17)	0.421(33)	1.18
$16^3 \times 32$	1.253(53)	0.404(94)	0.27
$2^{++} = a_1 \times D_- E$			
$12^3 \times 24$	1.597(93)	0.479(90)	0.12
$2^{--} = \rho \times D_- T_2$			
$12^3 \times 24$	1.769(270)	0.789(386)	2.32
$2^{--} = a_1 \times \nabla_- T_2$			
$16^3 \times 32$	1.508(24)	0.588(50)	1.67
$2^{--} = a_1 \times \nabla_- E$			
$16^3 \times 32$	1.472(28)	0.681(57)	5.88
$2^{-+} = \pi \times D_- T_2$			
$16^3 \times 32$	1.668(240)	0.480(375)	0.20
$3^{++} = a_1 \times D_- A_2$			
$16^3 \times 32$	1.859(95)	0.544(179)	0.41
$3^{--} = \rho \times D_- A_2$			
$16^3 \times 32$	1.746(26)	0.345(50)	2.16
$1^{-+} = b_1 \times \nabla_- T_1$			
$16^3 \times 32$	1.847(83)	0.421(162)	2.20

TABLE IX: Numerical results of the chiral extrapolations of the different high spin meson channels.

FIG. 2: Effective mass plots for mesons from our fine lattice (with $\kappa = 0.1410, 0.1400, 0.1390, 0.1375$ from top to bottom). Both ground and excited states are shown, along with the $M \pm \sigma_M$ results (horizontal lines) from correlated fits to the corresponding time intervals. For the V channel, we show results for both operator combinations.

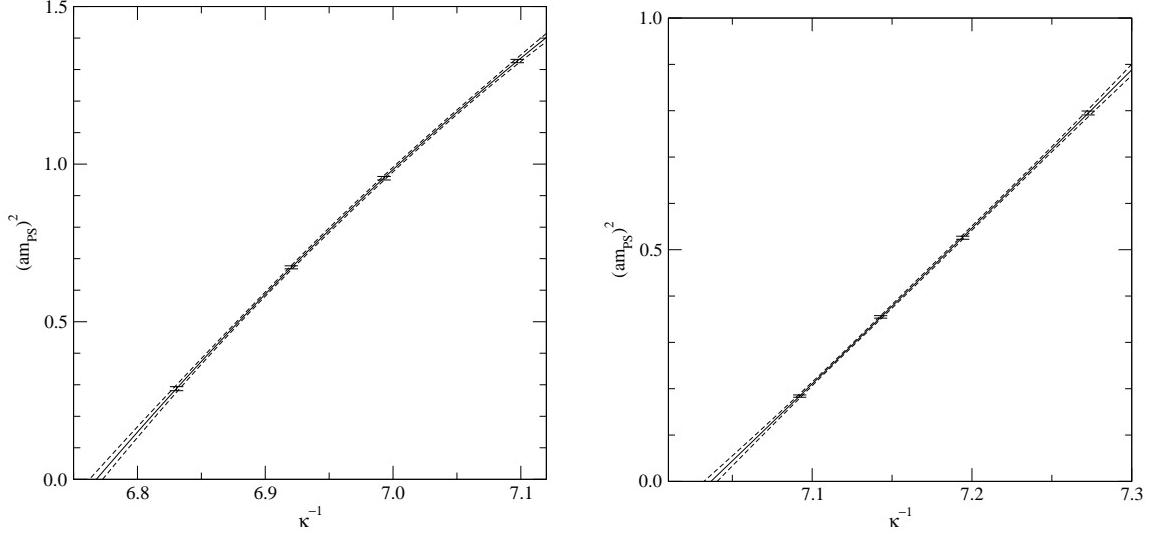


FIG. 3: The figure shows $(am_\pi)^2$ as a function of κ^{-1} . The left plot is for the $12^3 \times 24$ lattice, while the right plots shows the results for the $16^3 \times 32$ lattice.

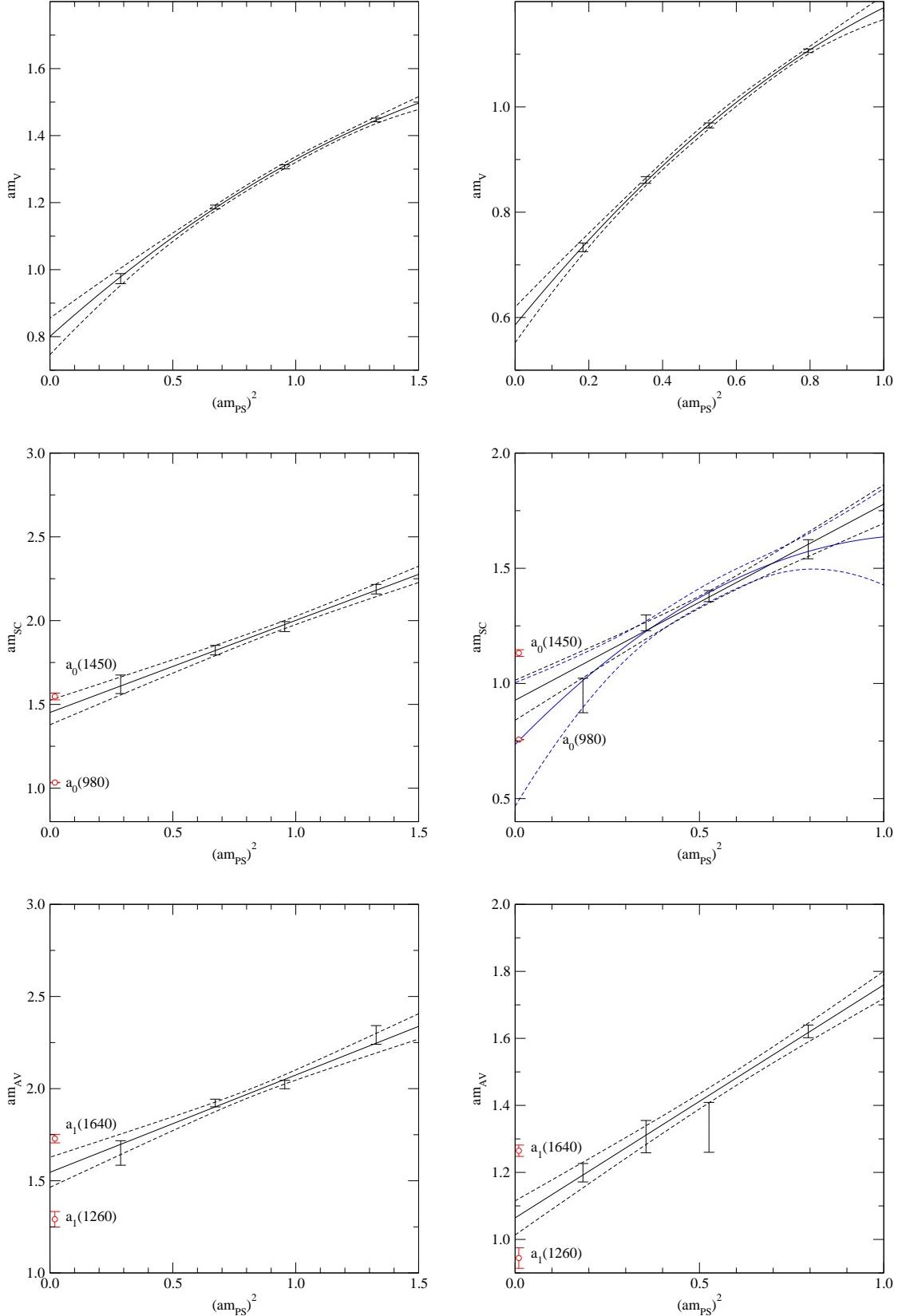


FIG. 4: The figure shows the vector, scalar, and axialvector meson ground states as a function of $(am_\pi)^2$. The left plots are for the $12^3 \times 24$ lattice, while the right plots show the results for the $16^3 \times 32$ lattice. The circles represent the experimental points. They are omitted for the vector meson since it is used to set the scale. We also show the results of our chiral extrapolation (solid line) together with the one sigma error band (dashed lines). For the scalar meson results on the $16^3 \times 32$ lattice both linear and quadratic fits have been performed.

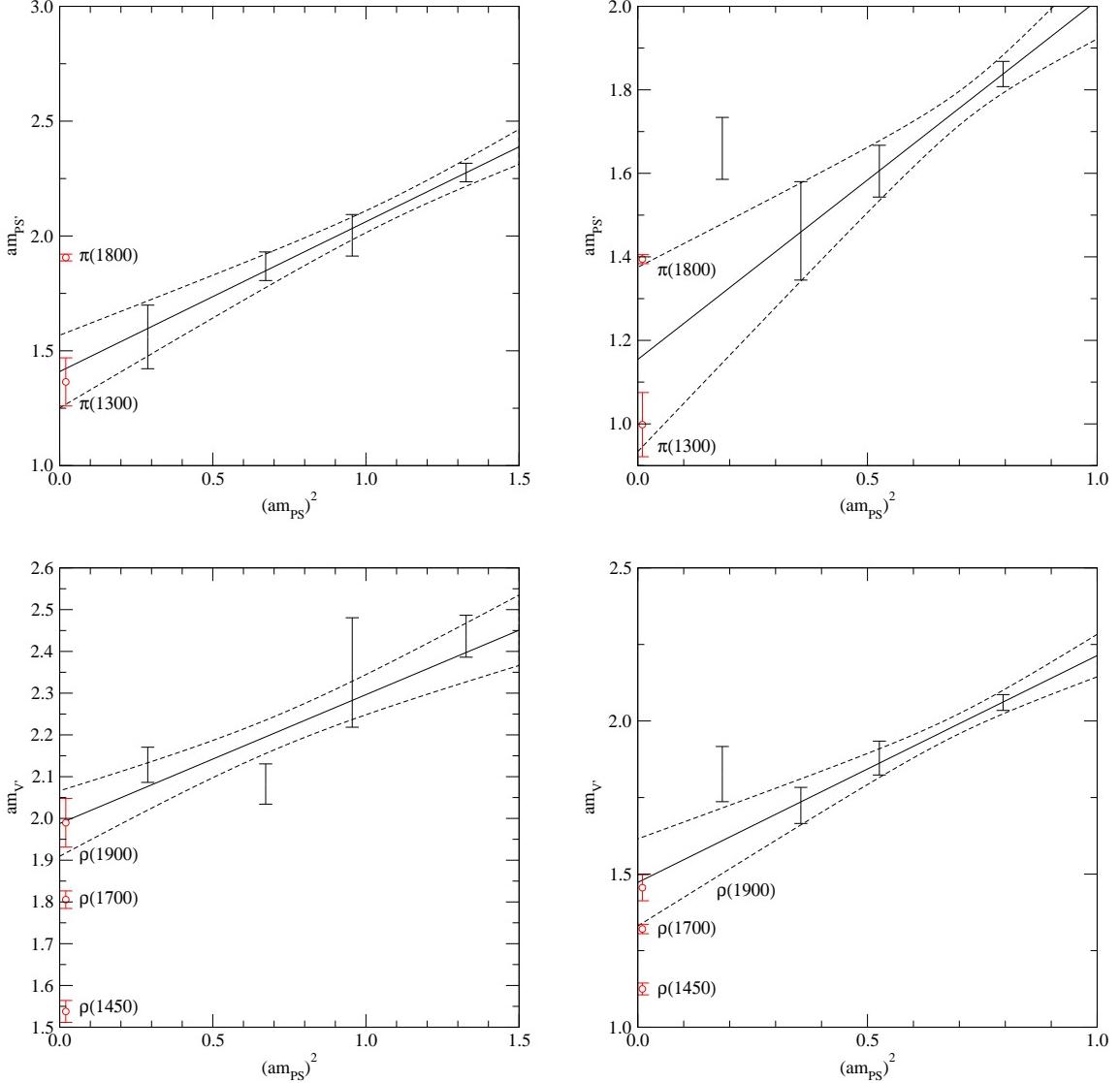


FIG. 5: The figure shows the pseudoscalar and vector meson first excited state as a function of $(am_\pi)^2$. The left plot is for the $12^3 \times 24$ lattice, while the right plot shows the results for the $16^3 \times 32$ lattice. The circles represent the experimental points. We also show the results of our chiral extrapolation (solid line) together with the one sigma error band (dashed lines).

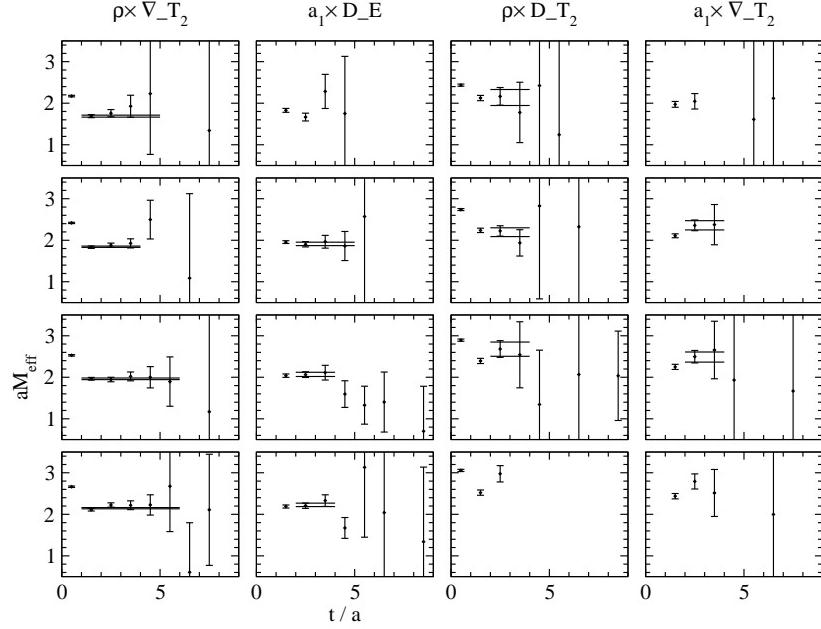


FIG. 6: Effective mass plots for the spin-2 meson interpolators $\rho \times \nabla \cdot T_2$, $a_1 \times D \cdot E$, $\rho \times D \cdot T_2$, $a_1 \times \nabla \cdot T_2$ from our coarse lattice (with $\kappa = 0.1464, 0.1445, 0.1430, 0.1409$ from top to bottom). Ground states are shown, along with the $M \pm \sigma_M$ results from correlated fits to the corresponding time intervals.

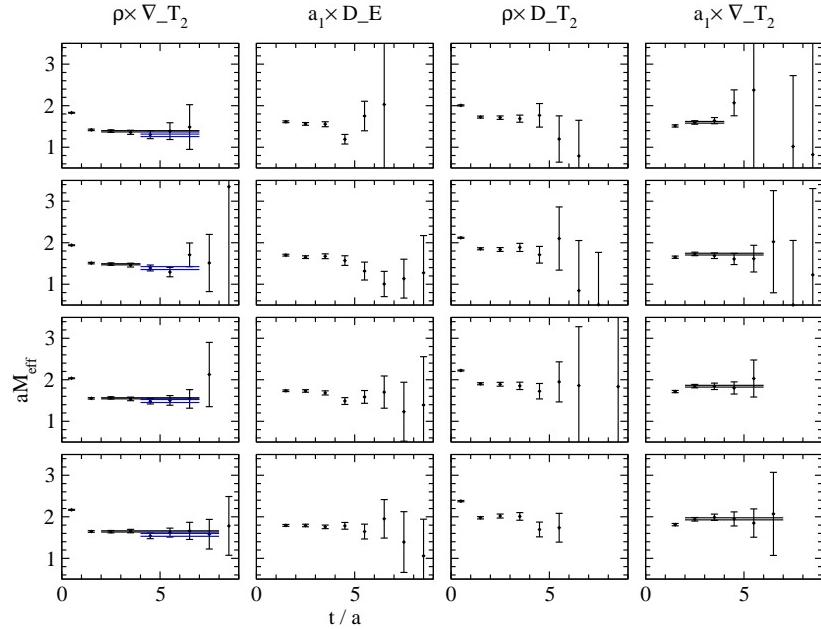


FIG. 7: Effective mass plots for the meson interpolators $\rho \times \nabla \cdot T_2$, $\rho \times D \cdot T_2$, $\rho \times D \cdot A_2$, $b_1 \times \nabla \cdot T_1$ from our fine lattice (with $\kappa = 0.1410, 0.1400, 0.1390, 0.1375$ from top to bottom). Ground states are shown, along with the $M \pm \sigma_M$ results from correlated fits to the corresponding time intervals. For the $\rho \times \nabla \cdot T_2$ channel, we show results from two fitting ranges.

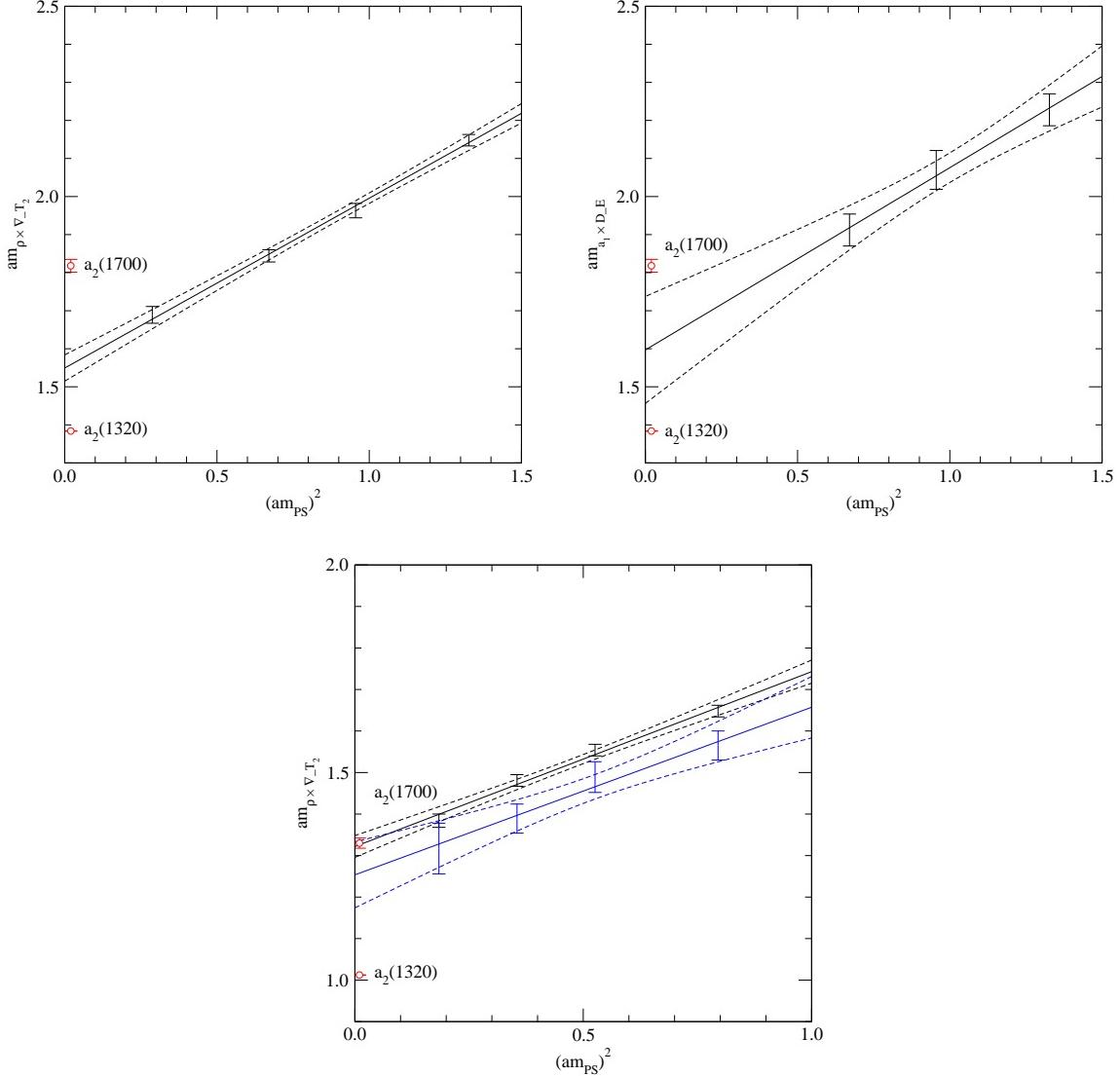


FIG. 8: The figure shows the ground states for our interpolators coupling to $J^{PC} = 2^{++}$ as a function of $(am_{\pi})^2$. The upper plots are for the $12^3 \times 24$ lattice. They show the results for the interpolators $\rho \times \nabla_- T_2$ (left-hand side) and $a_1 \times D_E$ (right-hand side). The lower plot is for the interpolator $\rho \times \nabla_- T_2$ on the $16^3 \times 32$ lattice and shows results for two different fit ranges of the correlator (see Fig. 7). The circles represent the experimental points. We also show the results of our chiral extrapolation (solid line) together with the one sigma error band (dashed lines).

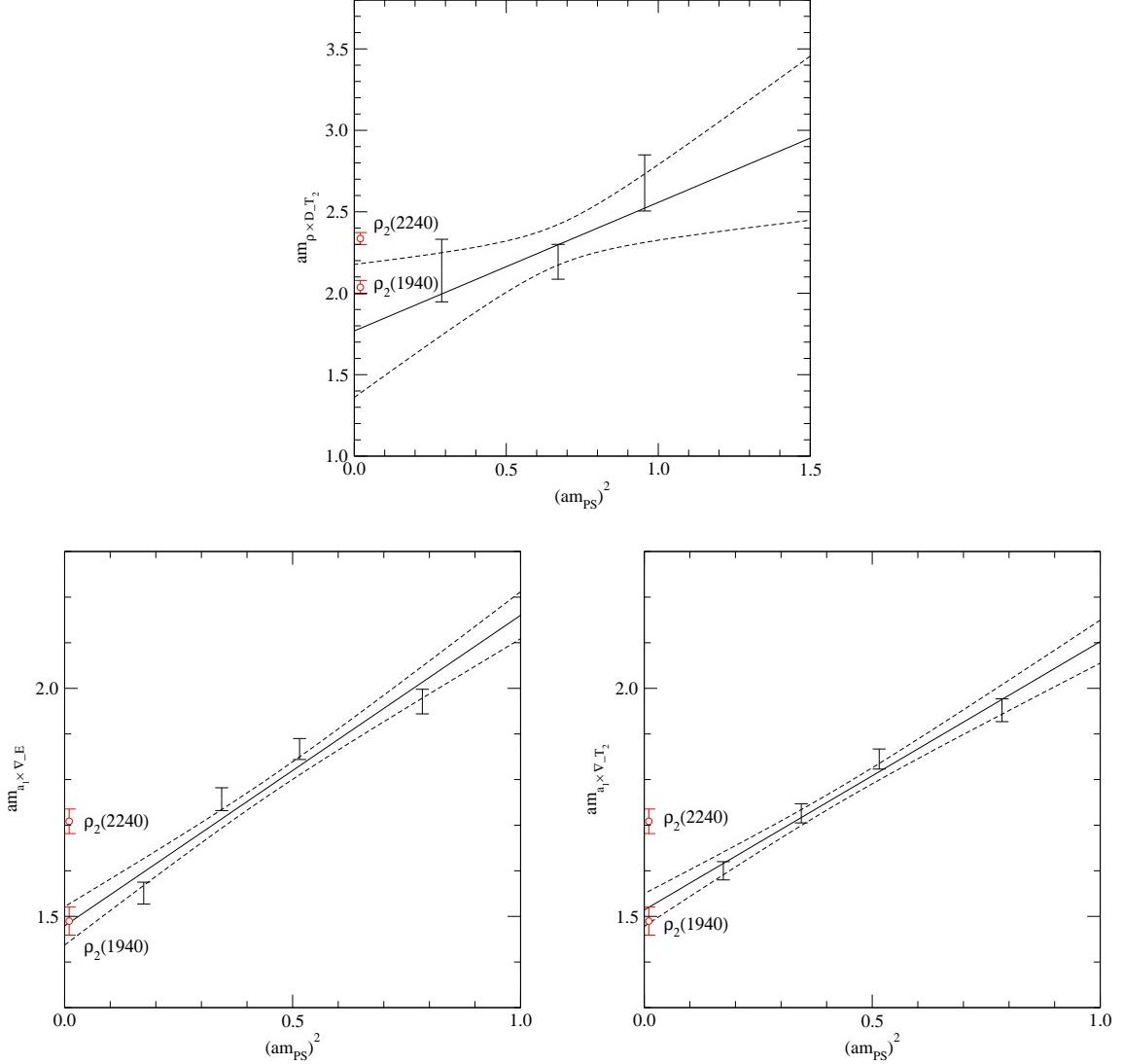


FIG. 9: The figure shows the ground states for our interpolators coupling to $J^{PC} = 2^{--}$ as a function of $(am_\pi)^2$. The upper plot is for the $12^3 \times 24$ lattice. It shows the results for the interpolator $\rho \times D_T2$. The lower plots are for the interpolators $a_1 \times \nabla_E$ (left-hand side) and $a_1 \times \nabla T_2$ (right-hand side) on the $16^3 \times 32$ lattice. The circles represent the experimental points. We also show the results of our chiral extrapolation (solid line) together with the one sigma error band (dashed lines).

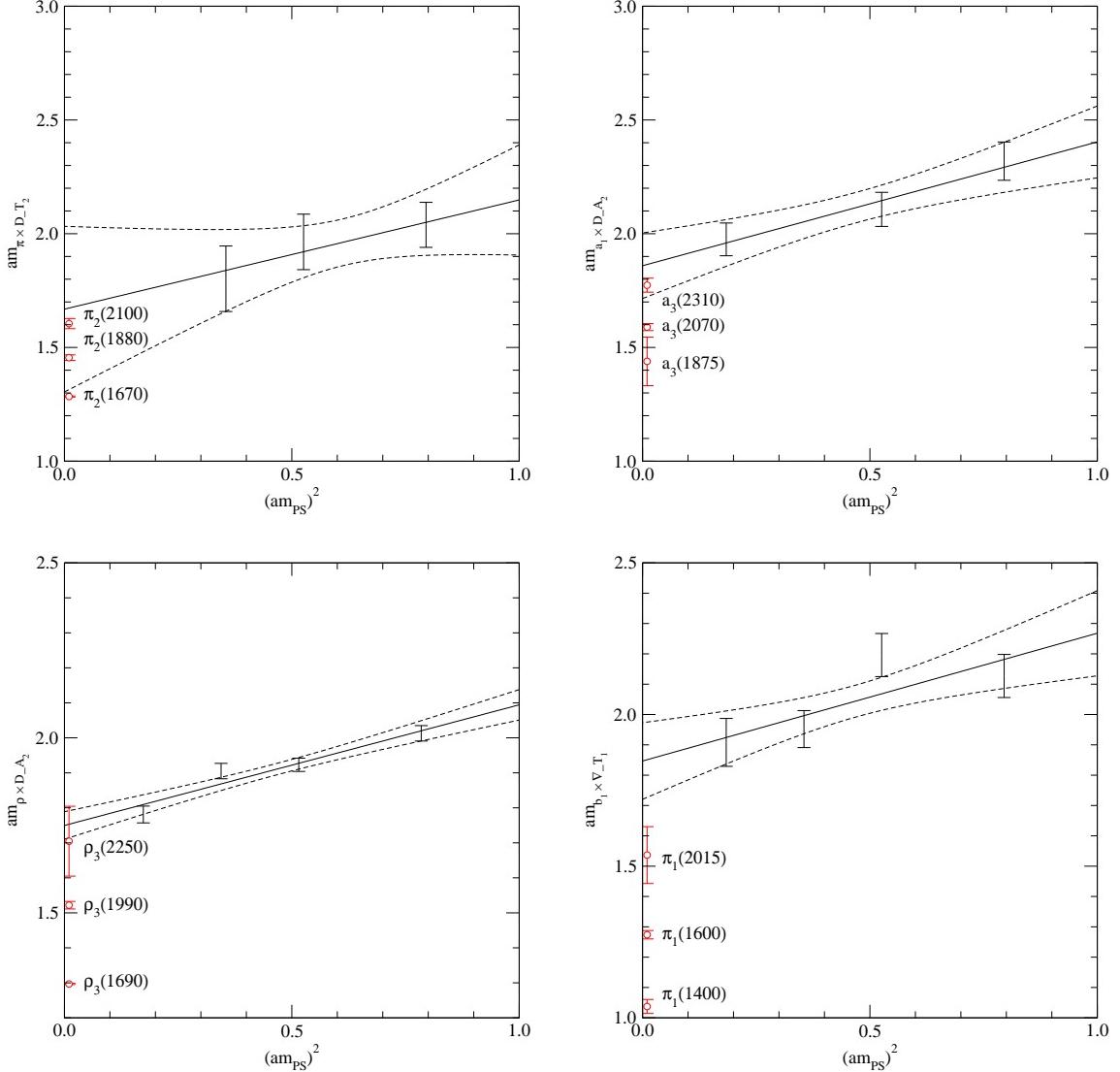


FIG. 10: The figure shows the ground states for interpolators which should couple to $J^{PC} = 2^{-+}, 3^{++}, 3^{--}, 1^{-+}$ as a function of $(am_\pi)^2$. The quality of our data allows us to show only results for the $16^3 \times 32$ lattice for the interpolators $\pi \times D_T$, $a_1 \times D_A$, $\rho \times D_A$ and $b_1 \times \nabla T_1$. The circles represent the experimental points. We also show the results of our chiral extrapolation (solid line) together with the one sigma error band (dashed lines).

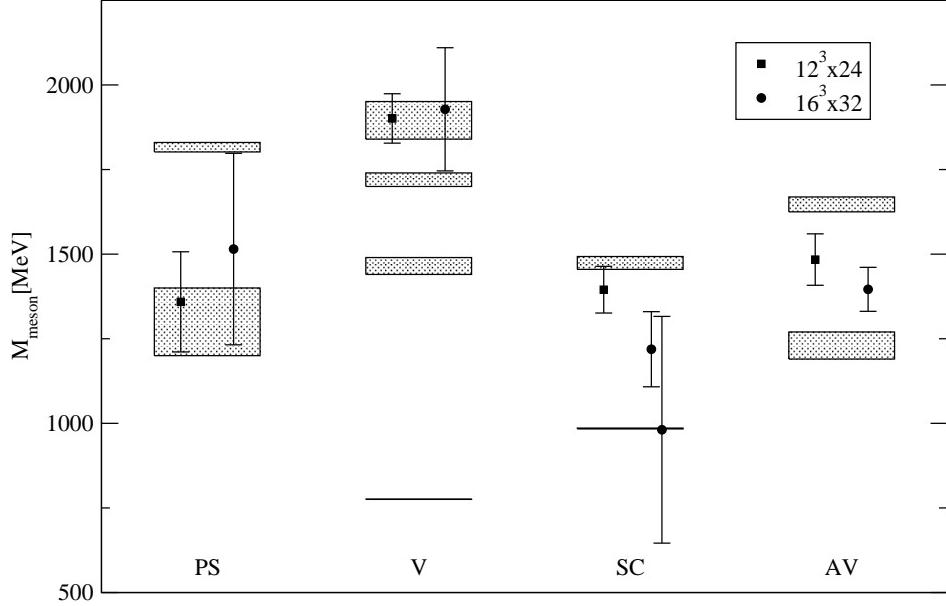


FIG. 11: Final results for the meson spectrum in the low spin sector. The boxes with the shaded areas represent the experimental values as classified by the Particle Data Group [36]. For the scalar meson on the fine lattice we present results both for linear and quadratic extrapolation in $(am_{PS})^2$. The vector meson and pseudoscalar meson ground states are not shown, since the former is used to fix the lattice spacing a , and the latter becomes massless in the chiral limit.

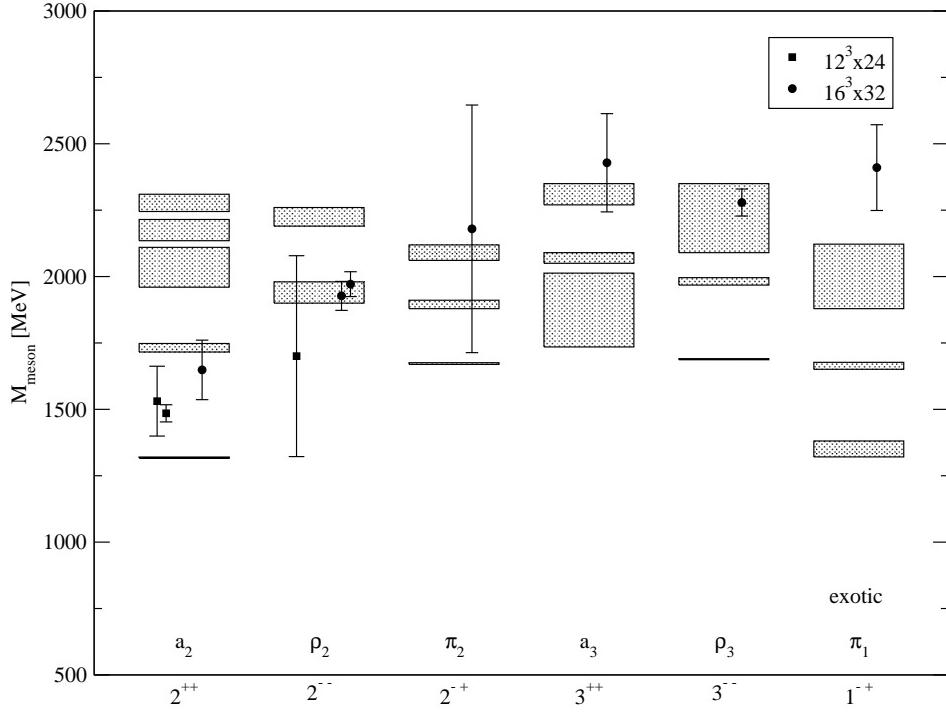


FIG. 12: Final results for the meson spectrum in the high spin and exotic sector. The boxes with the shaded areas represent the experimental values as classified by the Particle Data Group [36]. For the a_2 meson on the coarse lattice and for the ρ_2 meson on the fine lattice we present results belonging to different operators. For the π_2 meson, spin $J = 3$ mesons and the exotic π_1 meson we only see signals on the fine lattice.

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